Decision-Making under Insufficient Information

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I would like to introduce myself….

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Decision making – what is it actually?

- *Decision making* is the process of making choices by identifying a decision, gathering information, and assessing alternative resolutions. (*Umass Dartmouth*)
- *Decision making* is simply the process of making a choice. (*McKinsey & company*)
- *Decision making* is the process of evaluating alternatives and selecting an action. (Attwood et al. 2024)
- *Decision-making process* is a series of steps one or more individuals take to determine the best option or course of action to address a specific problem. (TechTarget)
- *Decision making* is a cognitive process resulting in the selection of a belief or a course of action among several possible alternative options. (Wikipedia)

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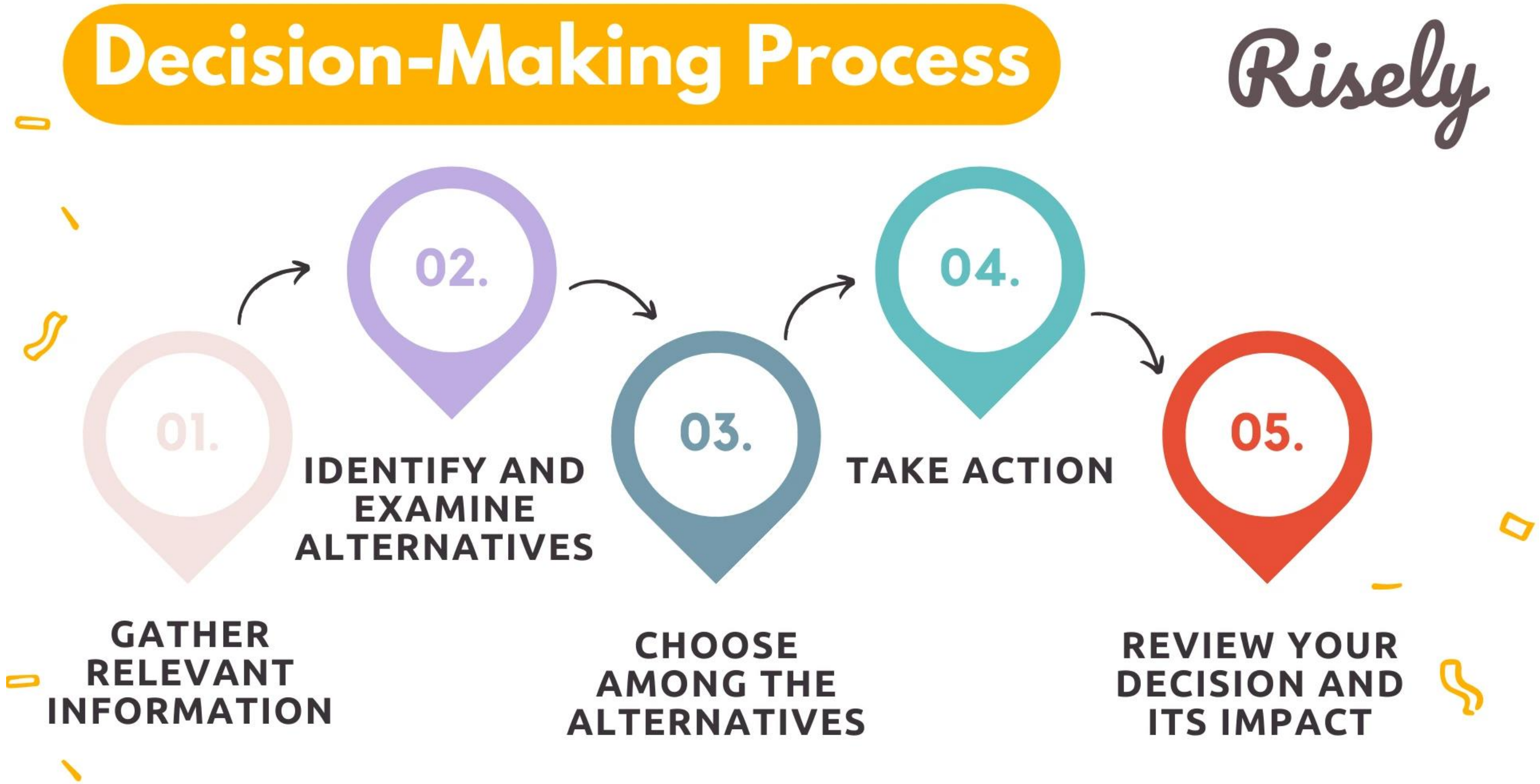
Decision making process



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Information

- According to the Merriam-Webster dictionary, *information* is:

a) Knowledge obtained from investigation, study, or instruction.

b) The attribute inherent in and communicated by one of two or more alternative sequences or arrangements of something (such binary digits in a computer program) that produce specific effects.

c) A signal or character (as in a communication system or computer) representing data.

d) Something (such as a message, experimental data, or a picture) which justifies change in a construct (such as a plan or theory) that represents physical or mental experience or another construct.

e) A quantitative measure of the content of information.

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The role of information on decision making

- Information plays a crucial role in the decision-making process and it serves as the foundation for informed decisions. The key ways in which information contributes to decision making: (*Ram, 2023*):
- *Problem Solving*: Information helps identify problems and allows decision makers to evaluate alternative solutions and choose the best course of action.
- *Evidence-based Decision Making*: Information provides the evidence needed to make decisions that are based on data and facts, rather than opinions or assumptions.
- *Risk Assessment*: Information helps decision makers assess risks and make decisions that minimize potential harm and maximize benefits.
- *Forecasting*: Information helps decision makers anticipate future events and trends, allowing them to make plans and prepare for potential challenges and opportunities.
- *Improved Accuracy*: Information helps decision makers make more accurate decisions by providing them with a complete and accurate picture of a situation.

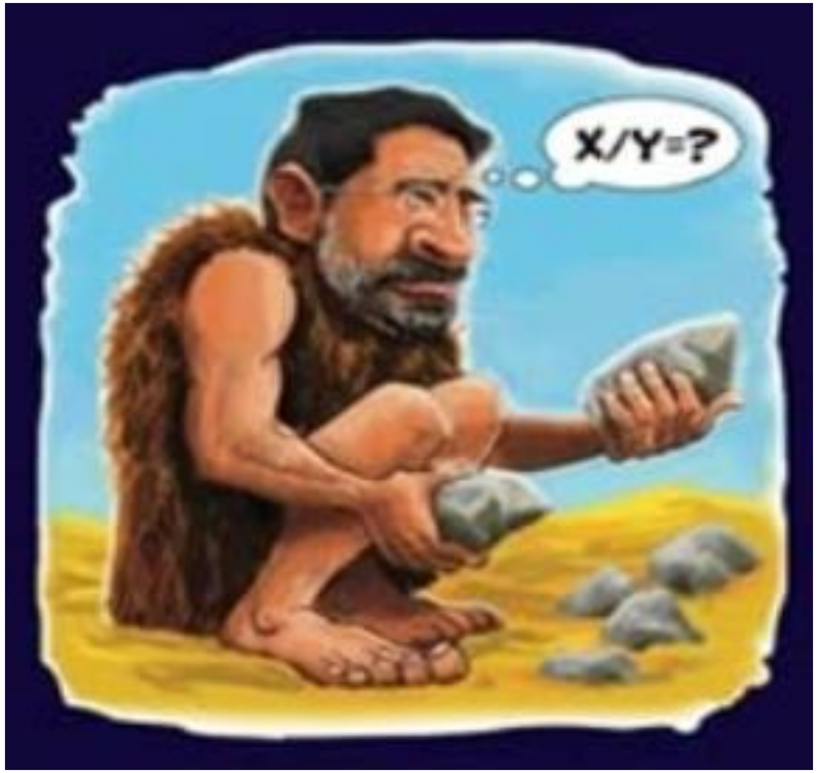
Decision making methods

- The goal of every decision problem is to select the best (most important, most preferred, etc.) alternative under given criteria. This is usually done by determining weights (priorities) of all alternatives.
- There are many methods for determining these weights (Odu, 2019):

- *Point allocation method,*
- *Direct rating mehtod,*
- *Ranking method,*
- *Swing method,*
- *Pairwise comparisons method,*
- … etc.

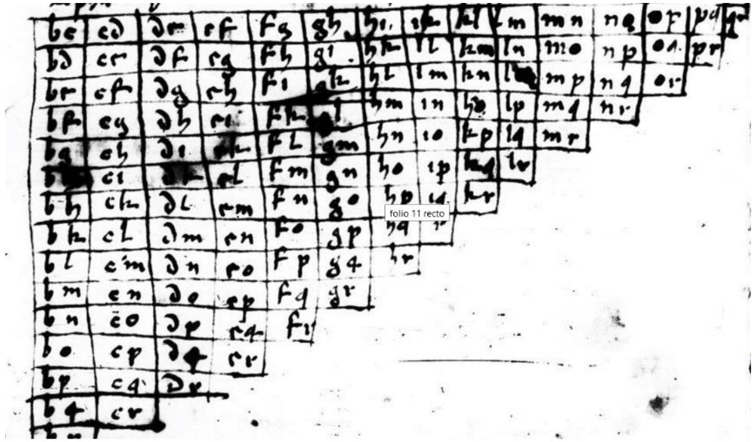
Pairwise comparison methods

- Pairwise comparisons belong among the oldest decision making methods in history (unknown Cavemen, 100,000 BC)…



Pairwise comparison methods

- Ramon Llull (Majorca c. 1232–1316), a Catalan monk and scholar, is considered one of the earliest founding fathers of voting theory and social choice theory.
- He is a first scholar who used pairwise comparisons in the treatise *Artifitium electionis personarum* in the context of an election of a prelate from 16 candidates who were all compared pairwise (see the figure below).



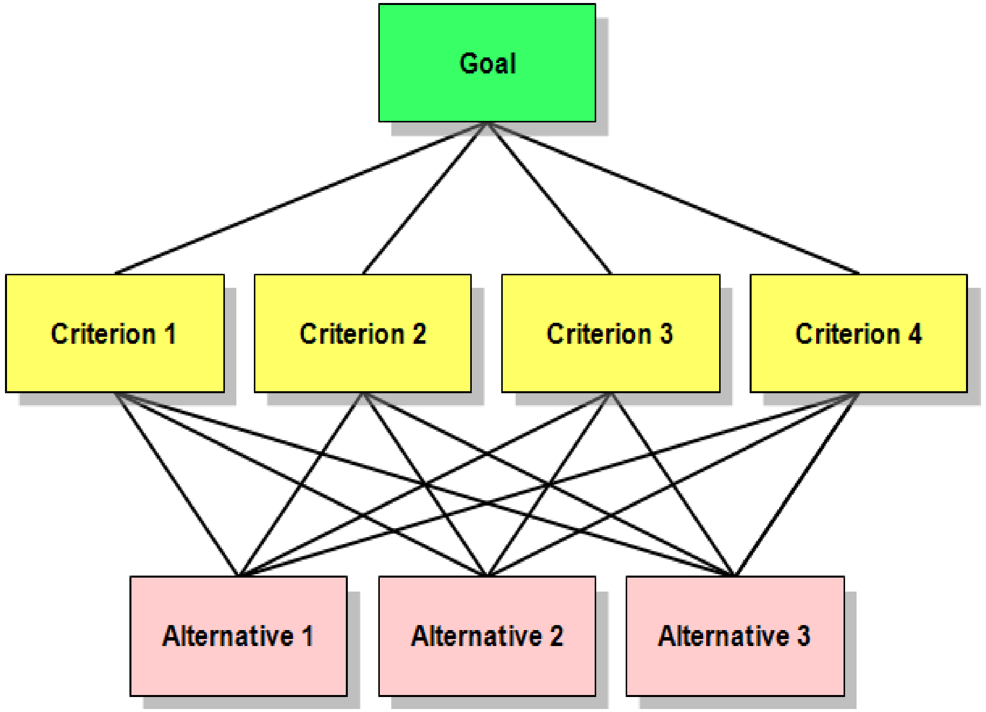
Pairwise comparison methods (cont.)

- We ask the following question: „How many times is one object better (more preferred, more important, etc.) than the second object?“
- For the comparison, we apply a suitable scale. The most common is Saaty’s scale from 1 to 9 (with reciprocals).
- For example, we can say that an apple is three times more preferred over a kiwi:

vs

Analytic Hierarchy Process (AHP)

- Proposed by T. L. Saaty in 1977.
- Multiple-criteria decision making method based on a hierarchical structure comprising Goal, Criteria and Alternatives.
- The comparison scale is from 1 to 9 (with reciprocals).
- All alternatives are compared pairwise with respect to every criterion, and all criteria are compared pairwise with respect to the goal.
- AHP enables to reduce decision making complexity.
- Applications of the AHP can be found in almost all areas of human action.



Analytic Hierarchy Process (cont.)

- All pairwise comparisons are arranged into a *pairwise comparisons matrix*:
- Weights (*w*) of all objects are calculated via the eigenvector method: $Aw = \lambda w$
- Alternatively, the weights can be derived via the geometric mean method.
- In the case of the „fruit example“ we get:
- w = (0.43, 0.33, 0.24).
- Hence, the apple has weight 0.43, the kiwi 0.33 and the pear 0.24.

			
	1	3	2
	1/3	1	2
	1/2	1/2	1

Analytic Hierarchy Process (cont.)

- A pairwise comparisons matrix $A = [a_{ij}], i, j \in \{1, \dots, n\}$ is *reciprocal*, if:

$$a_{ij} = \frac{1}{a_{ji}}, \forall\, i, j \qquad (1)$$

- A pairwise comparisons matrix $A = [a_{ij}], i, j \in \{1, \dots, n\}$ is *consistent*, if:

$$a_{ij} \cdot a_{jk} = a_{ik}, \forall\, i, j \qquad (2)$$

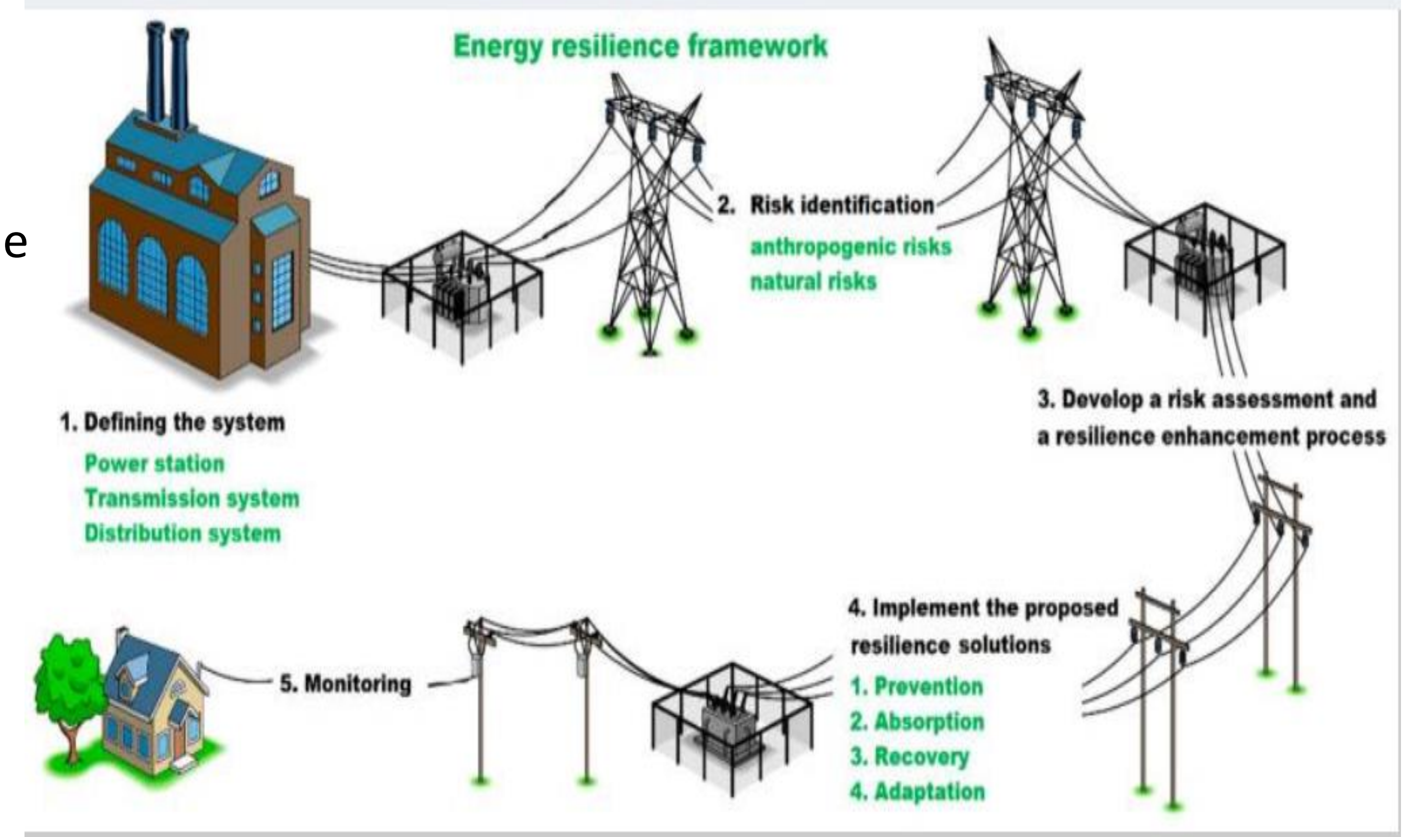
- *Consistency index* and *consistency ratio* of a matrix *A* are given as follows:

$$CI(A) = \frac{\lambda_{max} - n}{n-1}, CR(A) = \frac{CI(A)}{RI}, \qquad (3)$$

where λ_{max} denotes the principal eigenvalue of *A* and *RI* is the average *CI* of randomly generated matrices.

Energy resilience framework – the AHP application

- According to Energy resilience framework by (Phillips et al., 2016), the AHP can be applied, for example, in the risk assessment (step 3).

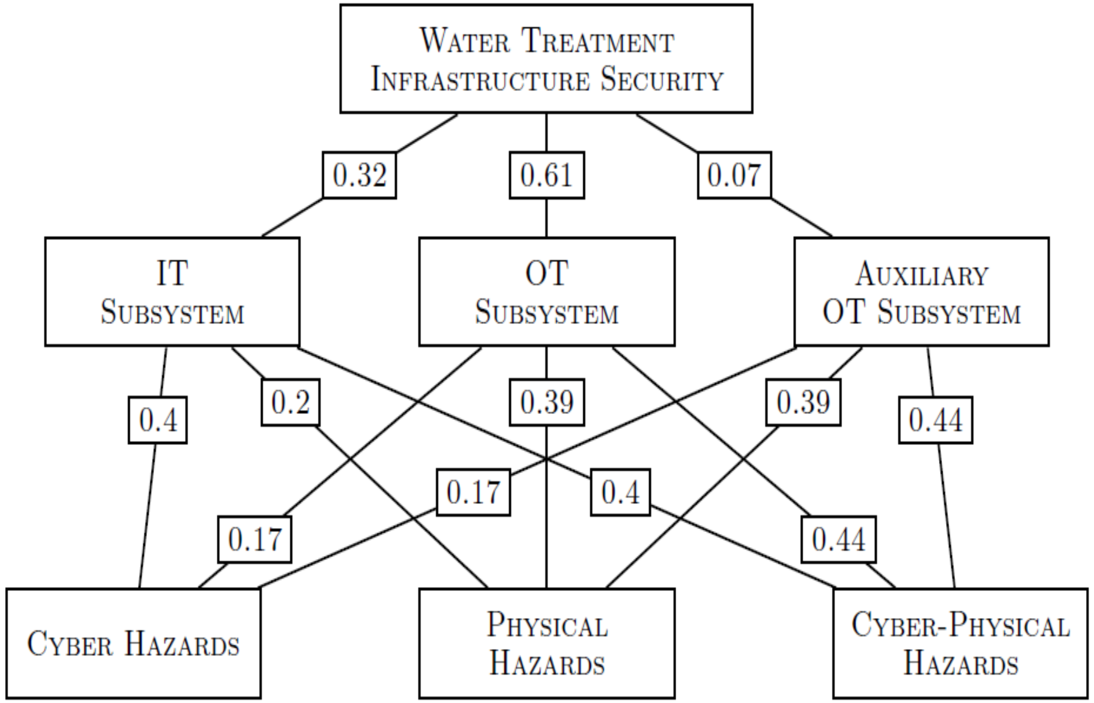


Credit: Phillips et al. (2016).

A Risk Assessment: an Example

- The study of (Fioravanti et al, 2022) defined the following hazard classes and attacks:

$\mathcal{A}_i^k$	Hazard class	Attack definition
$\mathcal{A}_1^1$	Cyber Hazards	Port Scan
$\mathcal{A}_2^1$	Cyber Hazards	Passive MITM
$\mathcal{A}_3^1$	Cyber Hazards	Phishing Campaign
$\mathcal{A}_4^1$	Cyber Hazards	Cross-Site Scripting
$\mathcal{A}_1^2$	Cyber Hazards	Code Injection
$\mathcal{A}_2^2$	Physical Hazards	Vandalism
$\mathcal{A}_3^2$	Physical Hazards	Explosion
$\mathcal{A}_4^2$	Physical Hazards	Hardware Failures
$\mathcal{A}_5^2$	Physical Hazards	Sabotage
$\mathcal{A}_1^3$	Cyber-Physical Hazards	Active MITM
$\mathcal{A}_2^3$	Cyber-Physical Hazards	DoS
$\mathcal{A}_3^3$	Cyber-Physical Hazards	DDoS



A Risk Assessment: an Example (cont.)

- Pairwise comparisons matrices of all attacks are shown on the right-hand side. (Fioravanti et al, 2022) . Vectors of weights are provided below.

$$A^1 = \begin{array}{c} \\ \mathcal{A}_1^1 \\ \mathcal{A}_2^1 \\ \mathcal{A}_3^1 \\ \mathcal{A}_4^1 \\ \mathcal{A}_5^1 \end{array} \begin{array}{c} \begin{array}{ccccc} \mathcal{A}_1^1 & \mathcal{A}_2^1 & \mathcal{A}_3^1 & \mathcal{A}_4^1 & \mathcal{A}_5^1 \end{array} \\ \begin{pmatrix} 1 & 1/2 & 2 & 1/2 & 1/3 \\ 2 & 1 & 3 & 1/2 & 1/2 \\ 1/2 & 1/3 & 1 & 1/4 & 1/5 \\ 2 & 2 & 4 & 1 & 1 \\ 3 & 2 & 5 & 1 & 1 \end{pmatrix} \end{array},$$

$$\mathbf{a}^1 = \begin{bmatrix} 0.12 \\ 0.18 \\ 0.07 \\ 0.30 \\ 0.33 \end{bmatrix}, \quad \mathbf{a}^2 = \begin{bmatrix} 0.09 \\ 0.33 \\ 0.25 \\ 0.33 \end{bmatrix}, \quad \mathbf{a}^3 = \begin{bmatrix} 0.16 \\ 0.30 \\ 0.54 \end{bmatrix}.$$

$$A^2 = \begin{array}{c} \\ \mathcal{A}_1^2 \\ \mathcal{A}_2^2 \\ \mathcal{A}_3^2 \\ \mathcal{A}_4^2 \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{A}_1^2 & \mathcal{A}_2^2 & \mathcal{A}_3^2 & \mathcal{A}_4^2 \end{array} \\ \begin{pmatrix} 1 & 1/4 & 1/3 & 1/4 \\ 4 & 1 & 2 & 1 \\ 3 & 1/2 & 1 & 1 \\ 4 & 1 & 1 & 1 \end{pmatrix} \end{array}, \qquad A^3 = \begin{array}{c} \\ \mathcal{A}_1^3 \\ \mathcal{A}_2^3 \\ \mathcal{A}_3^3 \end{array} \begin{array}{c} \begin{array}{ccc} \mathcal{A}_1^3 & \mathcal{A}_2^3 & \mathcal{A}_3^3 \end{array} \\ \begin{pmatrix} 1 & 1/2 & 1/3 \\ 2 & 1 & 1/2 \\ 3 & 2 & 1 \end{pmatrix} \end{array}.$$

Incomplete information (incomplete AHP)

- Sometimes, not all pairwise comparisons are available.
- This may happen due to time pressure of an expert, lack of knowledge, lack of empirical evidence, unreliable data, etc.
- In general, for *n* object we need at least (*n*-1) pairwise comparisons under the assumption that each object is compared at least once.
- Then, there are methods how to fill the missing comparisons.
- In this process, we apply the so called *consistency condition* (2): if an object A is 3 times better than an object B, and an object B is 2 times better than an object C, then A should be 6 times better than C.

Incomplete information (incomplete AHP)

- Consider the following pairwise comparisons matrix of four objects:

1	3	?	?
1/3	1	4	?
?	1/4	1	2
?	?	1/2	1

- In this matrix three pairwise comparisons above the main diagonal are missing.
- However, it is easy to fill the matrix via consistency condition: the first object is 3 times better than the second object and, simultaneously, second object is 4 times better than the third object. Therefore, the first object has to be 12 times better than the third object.

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Incomplete infomation (incomplete AHP)

- Hence, we obtain:

1	3	**12**	?
1/3	1	4	?
1/12	1/4	1	2
?	?	1/2	1

- In the same way, we can find the remaining two matrix elements.

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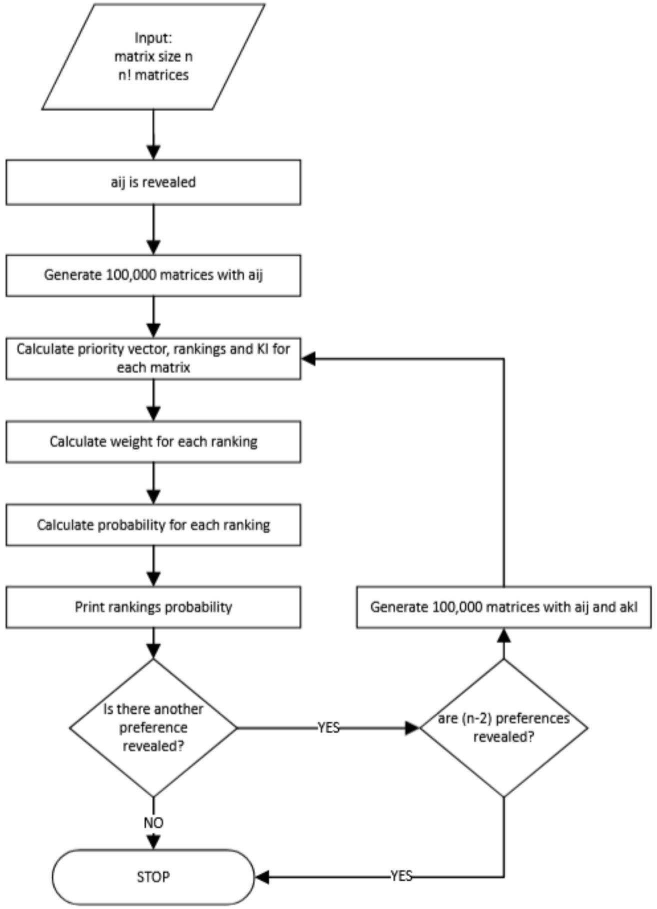
Decision making under insufficient information

- However, what if information provided by an expert is insufficient, that is less than (n-1) pairwise comparisons are available?
- This problem has never been considered in the literature and was regarded intractable.
- Nevertheless, there is a way how to handle such cases with the help of probability and simulation – the *Enumeration-Monte Carlo (EMC) algorithm*.
- The main idea of this approach is to simulate missing comparisons and assign probabilities to alternatives' rankings.
- To facilitate the use of the algorithm, the software tool was developed and is freely available at *https://github.com/RIIPCM/PerfMC.*

Enumeration-Monte Carlo Algorithm

- The algorithm is based on filling the missing matrix elements: all possible cases, when *n* is small, or 100,000 randomly generated cases, when *n* is large.
- Every filling is evaluated in terms of its consistency (3) and attains a weight depending on its (in)consistency.
- From every filling a ranking of objects is deduced.
- Finally, each ranking receives its final probability.



Decision making under insufficient information - Example

- Consider a situation where four objects (A, B, C and D) have to be ranked from the best to the worst on the basis of their pairwise comparisons.
- At the beginning, when no information is available, it is reasonable to consider all possible rankings (permutations) of A, B, C and D equally probable.
- Then, an expert steps in and reveals the first pairwise comparison: a_{12} = 5, which means A is 5 times better than B:

1	**5**		
1/5	1		
		1	
			1

Decision making under insufficient information - Example

- The EMC algorithm determines the probability of each ranking by randomly simulating the missing values.
- The most likely rankings with the probability of 9% are: ACBD, ADBC, CADB and DACB. In all of these rankings A is ranked better than B as one can expect since A is 5 better than B according to an expert.
- Next, another pairwise comparison is revealed by the expert: $a_{13} = 7$. This means C is 7 times better than A.

1	5	1/7	
1/5	1		
7		1	
			1

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Decision making under insufficient information - Example

- Now, the most likely rankings are CDAB (28.3%) followed by CADB (22.8%) and CABD (20.4%).
- As expected, in all three rankings A is ranked before B and C is ranked before A.
- Interestingly, since we have no information on D, D is ranked second, third and fourth respectively.
- The following Figure 1 shows how probability of rankings changed after the first and second pairwise comparison was revealed.
- Even with the insufficient information available its is possible to conclude that the object C is the most important.

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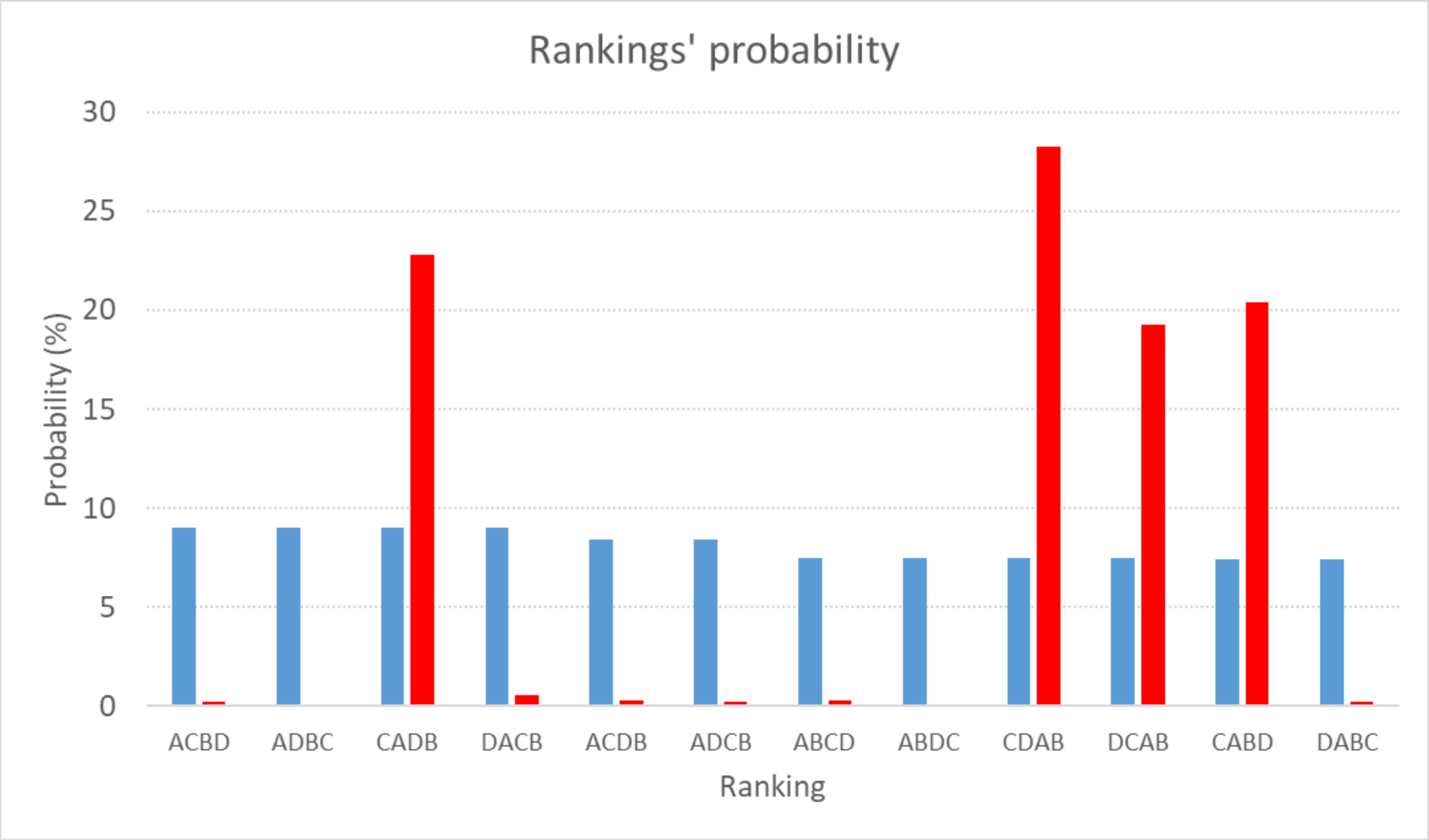
Decision making under insufficient information - Example



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Summary

- The aim of this presentation was to briefly introduce a newly developed decision making method (EMC algorithm) with insufficient information.
- For interested readers, short version of the algorithm was published here: https://ieeexplore.ieee.org/document/10480171.
- The full version of the paper is available on request.
- This pairwise comparisons method can be applied for example for a risk or hazard assessment, or essentially everywhere where a ranking (prioritization) is needed.
- As for insufficient information, during a war or a conflict, valuable information is usually scarce: an adversary intentionally conceals, distorts or obscures his actions, plans or intentions. But risks, hazards or dangers has to be estimated.
- For the method, a simple free online tool was developed and is available here: (*https://github.com/RIIPCM/PerfMC*).

THANK YOU FOR YOU ATTENTION

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